Problem Set 10 Discrete Mathematics Due on the 22nd of April, 2024

- (40 pts) 1. Construct—*with proof*—the *explicit* functions requested below.
 - (a) A bijection from $\{x \in \mathbb{R} \mid -1 < x < 1\}$ to $\{x \in \mathbb{R} \mid -\pi < x < \pi\}$.
 - (b) A surjection from \mathbb{N} to $\{p \mid p \text{ is prime}\}$.
 - (c) An injection from X to $\mathbb{P}(X)$ for *every* set X.
 - (d) A surjection from $\mathbb{P}(X)$ to X for *every* set $X \neq \emptyset$.
- (30 pts) 2. Let *A* be an arbitrary finite set of cardinality |A| = n, where $n \in \mathbb{N}$. How many finite strings over *A* are there?
- (30 pts)
 3. Imagine that, one day, you encounter a library.¹ At the entrance of this library is an enormous tome *B* listing *all* of the *possible* sentences in the English language, indexed by natural numbers.

Walking past the entrance, you see that the library has rows of bookshelves numbered 0, 1, 2, ..., so that there is exactly one row of books for each $n \in \mathbb{N}$. A great owl—perched on the pedestal that supports \mathscr{B} —informs you that, for each $n \in \mathbb{N}$, the n^{th} row of bookshelves contains *all* of the books that could possibly ever be that begin with the n^{th} sentence in \mathscr{B} . With the sound of granite scraping against marble, the doors to the library close behind you. The owl makes you the following proposal: you will be free to go *if and only if* you can read every book in the library *in a countable amount of time*.

Will you be set free? The owl demands a proof to justify your answer.

¹ This library is so massive and so complete a collection of literary works that it even contains books of *infinite length*.