## Problem Set 8

## Discrete Mathematics

Due on the $5^{\text {th }}$ of April, 2024
(10 pts) 1. Including the instructor, there are 32 people in our class. Prove that two of these people were born on the same day of the month.
(10 pts) 2. As of the $28^{\text {th }}$ of March, 2024, there are over 8.1 billion people living on Earth. ${ }^{1}$ A person's heart will beat no more than $7 \times 10^{9}$ times over their lifespan. Show that there are two currently-living people on Earth whose hearts have beat the exact same number of times.
(20 pts) 3. Let $n \in \mathbb{N}_{+}$and consider $\mathcal{A} \subseteq \mathbb{N}$ such that $|\mathcal{A}|=n+1$. Prove there exist $x, y \in \mathcal{A}$ with $x \neq y$ such that $n \mid x-y$.
(20 pts) 4. Consider $\mathcal{S}:=\{3,4,7,8,9,10,12,15,18,19,27,28\}$ and $\mathcal{X} \subseteq \mathcal{S}$ with $|\mathcal{X}| \geqslant 9$. Show that there exist three distinct elements $x_{1}, x_{2}, x_{3} \in \mathcal{X}$ such that $x_{1}+x_{2}+x_{3}=40$.
(20 pts) 5. Recall that $\binom{n}{0}=\binom{n}{n}=1$ for all $n, k \in \mathbb{N}$ when $k \leqslant n$.
(a) Show $\binom{n}{k}=\binom{n}{n-k}$ for all $n, k \in \mathbb{N}$ where $k \leqslant n$.
(b) Show $\binom{n+1}{k+1}=\binom{n}{k+1}+\binom{n}{k}$ for all $n, k \in \mathbb{N}$ where $k \leqslant n$.
(20 pts) 6. Prove that $|\mathbb{P}(X)|=2^{|X|}$ for any finite set $X$.
${ }^{1}$ For simplicity, you may assume the human population will only monotonically increase over time from the point this problem set was assigned.

