## Problem Set 6

## Discrete Mathematics

Due on the $5^{\text {th }}$ of March, 2024

All basic arithmetic and algebraic facts about $\mathbb{N}$ and $\mathbb{Z}$ are now yours to use.
(20 pts) 1.(a) Show that $(c \neq 0 \wedge a c \mid b c) \Rightarrow(a \mid b)$ for all $a, b, c \in \mathbb{Z}$.
(b) Show that $(n|x \wedge n| y) \Rightarrow(n \mid a x+b y)$ for all $n, x, y, a, b \in \mathbb{Z}$.
(20 pts) 2. For all $z \in \mathbb{Z}$, show that $z$ is even implies $z$ is not odd.
(20 pts) 3.(a) For all $n \in \mathbb{N}$, show that $n$ is even implies $n+1$ is odd.
(b) For all $n \in \mathbb{N}$, show that $n$ is odd implies $n+1$ is even.
(20 pts) 4. Show that $3 \mid n^{3}-n$ for all $n \in \mathbb{N} .{ }^{1} \quad{ }^{1}$ Hint: try a proof by induction.
(20 pts) 5. The Fibonacci sequence is the recursive function $\mathcal{F}: \mathbb{N} \rightarrow \mathbb{N}$ below.

$$
\begin{aligned}
\mathcal{F}(0) & :=0 \\
\mathcal{F}(1) & :=1 \\
\mathcal{F}(n+2) & :=\mathcal{F}(n+1)+\mathcal{F}(n)
\end{aligned}
$$

Show that $1+\sum_{i=0}^{n} \mathcal{F}(i)=\mathcal{F}(n+2)$ for all $n \in \mathbb{N}$.

