

## Problem Set 1

### Discrete Mathematics

Due on the 28<sup>th</sup> of January, 2024

Justify each of your answers with an argument.<sup>1</sup>

- (50 pts) 1. Determine the truth value of each sentence below.
- (a) "Madrid is the capital of Spain."
  - (b) "Santa Claus lives on the north pole."
  - (c) "This sentence is *false*."
  - (d) "The set of all sets that don't contain themselves contains itself."<sup>1</sup>
  - (e) "Red is a beautiful color."
  - (f) "Every declarative sentence is either *true* or *false* but not both."
  - (g) "If this sentence is *false*, then 7 is a prime number."<sup>2</sup>
  - (h) "The set of all sets contains itself."
  - (i) "This sentence is *true*."
  - (j) "If this sentence is *true*, then 2 is an odd number."<sup>3</sup>

<sup>1</sup> An answer provided with bad or no justification is as good as a wrong answer. Think carefully, and think deeply.

<sup>1</sup> A *set* is a collection of objects. When we talk about "*the set of all  $x$  with a property*," we mean the collection of all those  $x$  that have that property and *only* those  $x$ .

<sup>2</sup> Note that 7 really is a prime number.

<sup>3</sup> Note that 2 is not really an odd number.

- (25 pts) 2. Suppose we have an infinite sequence of sentences

$$S_0, S_1, S_2, \dots, S_i, \dots$$

where each sentence asserts that every sentence following it is *false*.

$$S_i := "S_j \text{ is } \textit{false} \text{ for all } j > i."$$

In this definition,  $i$  ranges over all of the natural numbers  $0, 1, 2, \dots$

What are the truth values of the sentences in this sequence?

- (25 pts) 3. In the sentence below, "*you*" refers to *you*, the student reading these sentences and solving this problem set. Determine the truth value of the following sentence.

"You have finitely many beliefs."